

## A STUDY ON CRITICAL THINKING FOR LEARNING AND TEACHING IN PROBABILITY AND STATISTICS

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### ABSTRACT

Critical thinking is very important to the cultivation of innovative talents. This paper firstly presents the definition of critical thinking, and then explains the application of critical thinking in the course of probability theory and mathematical statistics. Secondly, it shows the improvement of knowledge system by critical thinking. Finally, it discusses the cultivation of critical thinking in the classroom learning and teaching of probability theory and mathematical statistics.

**KEYWORDS:** Innovation, Critical thinking, Probability, Statistics

### 1. INTRODUCTION

Nowadays, the rapid development of society puts forward higher requirements for higher education, and people pay more and more attention to cultivating innovative talents who can adapt to the development of modern society and have multiple application abilities and can solve a variety of complex practical problems. Probability theory and mathematical statistics is a compulsory course for science and engineering majors in colleges and universities. It plays an important role in establishing random concepts for students, processing data problems and cultivating innovative thinking. However, in the process of classroom teaching, it is generally found that students' thinking is not active enough. Some students just think about how to pass the course without in-depth thinking. Some students plagiarize their classmates' wrong answers without analysis in order to cope with homework. Some students mechanically accept textbook knowledge and lack critical thinking ability. The so-called critical thinking, generally speaking, is to reflect on or question the problem with a critical eye. Research shows that critical thinking is one of the important factors of mathematical innovation. In recent years, the study of critical thinking has attracted much attention. Zhao (2016) discussed the application of critical thinking in linear algebra classroom teaching in determinant calculation and other aspects, and proposed several teaching methods of critical thinking cultivation. Yang (2016) studied how to strengthen the cultivation of critical thinking in higher mathematics teaching. It is emphasized that teachers should try to conducive the cultivate ideas, conditions and methods in teaching about critical thinking from many aspects. Wen (2018) introduces the spirit of questioning in critical thinking, the test of solving problems and other important contents from three perspectives. Chen (2017) studies the cultivation of critical thinking and innovative talents. It elaborates on taking various measures to cultivate critical thinking of college students. This paper presents the content of critical thinking contained in the teaching of

probability and mathematical statistics. Then it expounds how to strengthen the cultivation of critical thinking in the classroom learning and teaching of probability theory and mathematical statistics. So that students can better master various courses.

## 2 THE DEFINITION OF CRITICAL THINKING

At present, there are many definitions of critical thinking, among which the most comprehensive one is that critical thinking refers to the thinking that raises questions about a certain thing, reality and proposition, carries out reasonable and reflective analysis, and logically makes propositions based on self-selection and judgment (Yang 2016). While mathematical critical thinking is the personality quality of purposeful and conscious self-regulatory analysis, judgment, interpretation, reasoning and adjustment of existing mathematical expression, mathematical thinking process and results in the process of mathematical learning (Wen 2018). It includes the process of reflecting on and summarizing the thinking of oneself and others, raising questions, understanding the situation and analyzing independently.

For mathematical problems, we are often not satisfied with trivial and correct solutions, but appreciate simple, symmetrical and other beautiful results. The pursuit of multiple solutions and strict knowledge system is an important symbol of critical thinking, which promotes the continuous development and improvement of mathematics.

## 3 MULTIPLE SOLUTIONS TO ONE PROBLEM

There are a lot of problems in probability theory that can be solved in a variety of ways. This section outlines the widespread use of critical thinking with the expression of random events and the calculation of correlation coefficients. Two examples are addressed as follows.

Example 1. Let A, B and C are three events, and express that A, B and C are no more than two occur.

This topic can be expressed in three different ways:

① The opposite of what's going on  $\overline{ABC}$ .

② At least one didn't happen:  $\overline{A} \cup \overline{B} \cup \overline{C}$ .

③ Either exactly 2 of them happen, or exactly 1 of them happen, or none of them happen:  
 $AB\overline{C} \cup A\overline{B}C \cup \overline{A}BC \cup A\overline{B}\overline{C} \cup \overline{A}B\overline{C} \cup \overline{A}\overline{B}C \cup \overline{A}\overline{B}\overline{C}$ .

Obviously, the expressions in ① and ② are simpler and satisfy DE Morgan law. But the expression in ③ is cumbersome. However, three methods can help students fully understand the meaning of the event.

**Example 2.** Flip a coin (with heads and tails)  $n$  times in a row.  $X$  and  $Y$  represent the number of heads and the number of tails respectively. Find the correlation coefficients of  $X$  and  $Y$ .

When this problem is raised in class, most students will think of a solution formula that can use the correlation coefficient

$$\rho_{XY} = \frac{Cov(X, Y)}{\sqrt{D(X)} \sqrt{D(Y)}} \quad (1)$$

However, considering formula (1), it is troublesome or even impossible to get the answer. Therefore, some students tried to find an easier solution. The meaning of the question can be obtained by  $X+Y=n$ . This means that  $Y=-X+n$ . By using the property of correlation coefficient, we can directly deduce that the correlation coefficient is -1 between  $X$  and  $Y$ , which is both fast and good. This is the embodiment of critical thinking.

In a word, students should learn to use critical thinking to solve problems. They should try to take multiple methods for the same problems. At the same time, many formulas, proofs and algorithms will be questioned in the learning process. In the long run, his thinking ability and innovative spirit will be improved a lot.

## 4 CONTINUOUS REVISION FOR THE KNOWLEDGE SYSTEM

Critical thinking plays an important role in the improvement and development of statistical knowledge system. The following unknown parameter moment estimation and right hypothesis test are the best examples which are continuous improvement through development.

### 4.1 The Unknown Parameter Moment Estimation

About 1990s, the engineering statistics textbooks introduce to estimate the unknown parameter moment based on the law of large Numbers: "the sample  $K$ -order origin moment converges to the total  $k$ -order origin moment in probability". At that time, in order to solve the unknown parameters, let the sample  $K$ -order origin moment be equal to the population  $K$ -order origin moment. The moment estimation of unknown parameters seems to be too rigid and forced. So the present textbook uses the approximate substitution method. A more subtle approach makes it easier for the readers to understand and accept.

### 4.2 The Right Hypothesis Test Problem

In statistics, for the problem of right hypothesis test  $H_0: \mu \leq \mu_0, H_1: \mu > \mu_0$ , about thirty years ago, it is done according to the situation  $H_0: \mu = \mu_0, H_1: \mu > \mu_0$ . Obviously, that is not strict enough. In the last ten years, because of the criticism, discussion and pursuit of experts and scholars, the textbook

(Sheng 2008) uses a more systematic and more accurate method to test which is convincing. Therefore, critical thinking is one of the fundamental motivations to promote the continuous development and improvement of mathematical knowledge system.

## **5 THE CULTIVATION OF CRITICAL THINKING**

With the continuous development of quality education and educational innovation in today's society, people pay more and more attention to critical thinking, so how to cultivate critical thinking becomes particularly important. Richard Levin, former President of Yale University, pointed out pointedly that China's undergraduate education lacks the cultivation of critical thinking (Chen 2017). This deserves our deep reflection and introspection. Critical thinking requires teachers and students to cooperate with each other and build a new type of fair, equal and mutually promoting teacher-student relationship (Zhao 2016). The following suggestions are put forward.

### **5.1 College Students Should Consciously Improve Their Innovation Consciousness and Critical Thinking Ability.**

For a long time, how to analyze, solve and apply mathematical problems has been regarded as the key to learn probability theory and mathematical statistics well. Students always learn knowledge based on textbooks instead of associating with practical applications, which leads to their inflexible thinking and loss of interest in knowledge. Therefore, for college students, they should have a deep understanding of the nature of new knowledge. Then, they should know what it is and how it works. For the problem of probability and statistics, we should not only learn to draw inferences from one example, but also associate with practical application and consciously cultivate critical thinking.

### **5.2 Teachers Should Encourage Students to Question.**

Questioning is a kind of psychological activity that pushes people to pursue knowledge in the form of solving specific questions. Questioning is the source of thinking and the internal driving force of learning. It can change students' thirst for knowledge from latent state to active state. Without reasonable doubt, there is no critical thinking. Therefore, encouraging students to question is an important part of cultivating students' critical thinking (Yang 2016). Asking students to climb on the blackboard before class is a good way to develop critical thinking. After the students finishing the questions in front of the blackboard, the teacher comments on the wrong questions and extends them, which can broaden the students' thinking.

In a word, teachers should encourage students to love knowledge and science. Teachers and students jointly pursue better solutions to probability and statistics problems, cultivate critical thinking awareness, and constantly provide high innovation ability.

## **6 SUMMARY AND ACKNOWLEDGEMENT**

Critical thinking is the habit of thinking critically about the opposite of what you are facing. In the teaching of probability theory and mathematical statistics in colleges and universities (Sheng 2008),

critical thinking should be applied to classroom teaching to cultivate students' independent consciousness and innovative spirit. At the same time, teachers should actively guide students to use critical thinking to solve problems, and strive to cultivate batch after batch of top innovative talents with profound knowledge and critical thinking for the country.

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