

To cite this article: Duong Lam Thuy and Huong Dong Thi Mai (2026). DESIGN AND EVALUATION OF INSTRUCTIONAL STRATEGIES FOR DEVELOPING MATHEMATICAL REASONING COMPETENCY THROUGH FRACTIONS AND FRACTION OPERATIONS IN GRADE 4, International Journal of Education and Social Science Research (IJESSR) 9 (4): 325-342 Article No. 1274, Sub Id 1952

DESIGN AND EVALUATION OF INSTRUCTIONAL STRATEGIES FOR DEVELOPING MATHEMATICAL REASONING COMPETENCY THROUGH FRACTIONS AND FRACTION OPERATIONS IN GRADE 4

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DOI : <https://doi.org/10.37500/IJESSR.2026.9415>

ABSTRACT

Developing mathematical reasoning competency has become a central objective of competency-based mathematics education because it enables primary school students to explain mathematical ideas, justify solutions, and solve mathematical problems through logical reasoning. Fractions and fraction operations are among the most conceptually demanding topics in Grade 4 mathematics and provide important opportunities for fostering students' mathematical reasoning. This study investigated the current status of teaching fractions and fraction operations to develop mathematical reasoning competency and established an empirical foundation for designing instructional strategies. A mixed-methods research design was employed, including questionnaire surveys, classroom observations, interviews, and document analysis. The study involved 16 Grade 4 teachers and 205 Grade 4 students from two primary schools in Bac Ninh Province, Vietnam. The findings revealed that all participating teachers recognized the importance of developing mathematical reasoning competency, and most reported that they regularly integrated this objective into mathematics instruction. However, classroom practice remained largely focused on basic mathematical thinking and procedural problem solving, while opportunities for students to construct mathematical arguments and justify their reasoning were comparatively limited. Traditional instructional approaches were still widely adopted despite teachers' awareness of competency-based teaching. These findings demonstrate the need for systematic instructional strategies that support teachers in integrating mathematical reasoning into the teaching of fractions and fraction operations. The study provides practical evidence for improving competency-based mathematics instruction and promoting mathematical reasoning competency among Grade 4 students.

KEYWORDS: fractions; fraction operations; mathematical reasoning competency; Grade 4 mathematics; primary education

INTRODUCTION

Developing students' mathematical reasoning competency has become a fundamental objective of contemporary mathematics education because reasoning enables learners to explain mathematical relationships, justify conclusions, evaluate solution strategies, and construct logical arguments when solving mathematical problems. Rather than relying solely on procedural fluency, competency-based mathematics education emphasizes students' ability to think critically, communicate mathematical ideas, and apply reasoning across diverse learning contexts. Consequently, strengthening mathematical reasoning has become an important priority in curriculum reform and mathematics teaching worldwide. The National Council of Teachers of Mathematics (2014) identifies reasoning and justification as essential mathematical practices that should be systematically integrated into classroom instruction, while the OECD (2023) highlights mathematical reasoning as a key component of mathematical literacy required for solving real-world problems and making evidence-based decisions.

Among the mathematical topics taught in primary schools, fractions and fraction operations present particular challenges because they require students to develop conceptual understanding while simultaneously applying logical reasoning to compare quantities, justify procedures, and explain mathematical relationships. Learning fractions involves much more than performing calculations; students are expected to construct mathematical arguments, recognize equivalent representations, analyze relationships between quantities, and explain why specific procedures are mathematically valid. As emphasized by English and Kirshner (2016), mathematical reasoning develops most effectively when students actively engage in meaningful mathematical discussions, explain their thinking, and justify their conclusions through carefully designed learning experiences. Likewise, Lesh and Zawojewski (2007) argue that reasoning and problem solving are inseparable processes through which students construct mathematical understanding and refine increasingly sophisticated mathematical models.

Despite the recognized importance of mathematical reasoning, previous studies have reported that classroom instruction continues to emphasize procedural knowledge more strongly than reasoning, justification, and mathematical communication. Teachers frequently encounter difficulties in designing instructional activities that encourage students to explain their thinking, evaluate alternative solutions, and construct mathematical arguments, particularly when teaching conceptually demanding topics such as fractions. Schoenfeld (2013) noted that effective mathematics instruction requires learning environments that deliberately cultivate reasoning, reflection, and mathematical argumentation rather than focusing primarily on obtaining correct answers. Similarly, Lester (2007) emphasized that developing mathematical reasoning requires coherent instructional designs capable of engaging students in progressively more complex reasoning processes throughout mathematics learning.

Against this background, the present study investigates the current status of teaching fractions and fraction operations for developing mathematical reasoning competency among Grade 4 students in Vietnamese primary schools. By examining teachers' perceptions, instructional practices, and existing

challenges, the study establishes an empirical foundation for designing instructional strategies that support competency-based mathematics teaching. The findings are expected to contribute practical evidence for improving classroom instruction while providing useful implications for curriculum implementation and teacher professional development in primary mathematics education.

2. LITERATURE REVIEW

2.1. Mathematical Reasoning Competency

Mathematical reasoning competency refers to an individual's ability to analyze mathematical situations, identify relationships, formulate logical arguments, justify conclusions using mathematical evidence, and evaluate the validity of different solution strategies. Within competency-based mathematics education, reasoning extends beyond obtaining correct answers and involves constructing explanations, making conjectures, comparing alternative approaches, and communicating mathematical thinking through coherent arguments. Consequently, mathematical reasoning is recognized as a core competency that enables learners to develop conceptual understanding while applying mathematical knowledge flexibly in unfamiliar situations.

According to Jeannotte and Kieran (2017), mathematical reasoning should be understood as a dynamic process involving the generation, validation, and communication of mathematical ideas rather than as a single cognitive skill. Their conceptual model emphasizes that reasoning encompasses multiple interconnected processes, including conjecturing, generalizing, justifying, proving, and evaluating mathematical claims. From this perspective, classroom instruction should provide students with opportunities to explain their thinking, construct mathematical arguments, critique different solution strategies, and reflect on the validity of mathematical conclusions. Such learning experiences enable students to develop deeper conceptual understanding while strengthening higher-order mathematical thinking.

In the context of primary mathematics education, developing mathematical reasoning competency is particularly important because it establishes the cognitive foundation for subsequent mathematical learning. Learning activities that encourage students to explain procedures, justify relationships among mathematical concepts, compare different solution methods, and evaluate mathematical arguments contribute significantly to the development of reasoning competency. Therefore, instructional design should deliberately incorporate opportunities for mathematical discussion, argumentation, and reflection throughout the teaching process, particularly when students learn conceptually demanding topics such as fractions and fraction operations.

2.2. Teaching Fractions and Fraction Operations

Fractions and fraction operations constitute one of the most conceptually demanding domains in primary mathematics because they require students to understand part-whole relationships, equivalence, comparison, and the meanings underlying mathematical operations rather than relying solely on computational procedures. Successful learning of fractions depends on students' ability to connect multiple mathematical representations, explain relationships among quantities, and justify the

procedures used to solve mathematical problems. Consequently, teaching fractions should emphasize conceptual understanding and mathematical reasoning simultaneously, enabling students to construct meaningful mathematical knowledge through active engagement with mathematical ideas.

According to Lamon (2012), effective instruction on fractions should move beyond memorization of rules and algorithms by providing students with rich learning experiences that promote conceptual understanding through reasoning, exploration, and discussion. Students should be encouraged to interpret fractions in different contexts, compare alternative solution strategies, explain mathematical relationships, and justify why particular procedures are valid. Such instructional approaches help learners develop flexible mathematical thinking while reducing common misconceptions associated with fraction concepts and operations.

Within competency-based mathematics education, teaching fractions therefore requires carefully designed instructional activities that integrate conceptual learning with reasoning processes. Authentic mathematical situations, visual representations, collaborative problem-solving, and classroom discussions provide opportunities for students to formulate mathematical arguments, evaluate different approaches, and communicate their reasoning logically. By embedding these learning experiences throughout fraction instruction, teachers can support not only students' mastery of fraction concepts and operations but also the progressive development of mathematical reasoning competency, which serves as a foundation for subsequent mathematical learning.

2.3. Instructional Strategies for Developing Mathematical Reasoning Competency

Developing mathematical reasoning competency requires instructional strategies that actively engage students in constructing, explaining, and evaluating mathematical ideas rather than simply applying memorized procedures. Competency-based mathematics instruction therefore emphasizes learning environments in which students formulate conjectures, justify their thinking with mathematical evidence, compare alternative solution methods, and critically evaluate the validity of mathematical arguments. Within such environments, teachers play the role of facilitators who design meaningful mathematical tasks, encourage dialogue, and guide students through progressively more sophisticated reasoning processes.

According to Stylianides and Stylianides (2009), instructional strategies that promote mathematical reasoning should provide systematic opportunities for students to move from intuitive or empirical explanations toward logically justified mathematical arguments. This transition is achieved by engaging learners in activities that require them to explain why a mathematical statement is true, evaluate the reasoning of others, identify misconceptions, and construct convincing mathematical justifications. Rather than emphasizing only correct answers, effective instruction should cultivate habits of questioning, argumentation, and reflection, enabling students to develop increasingly rigorous mathematical reasoning over time.

In the teaching of fractions and fraction operations, instructional strategies should therefore incorporate authentic mathematical situations, exploratory learning activities, collaborative discussion, and opportunities for students to communicate and defend their mathematical thinking. Teachers should encourage learners to explain relationships among fraction concepts, justify computational procedures, compare different solution strategies, and reflect on the validity of their conclusions. Such instructional practices not only strengthen students' conceptual understanding of fractions but also promote the continuous development of mathematical reasoning competency, thereby supporting the broader goals of competency-based primary mathematics education.

3.1. Research Design

This study adopted a mixed-methods research design to investigate the current status of developing mathematical reasoning competency through the teaching of fractions and fraction operations in Grade 4 and to establish an empirical basis for designing instructional strategies. The research integrated quantitative and qualitative approaches to obtain comprehensive evidence regarding teachers' perceptions, classroom practices, and instructional challenges. Quantitative data were collected through questionnaire surveys administered to teachers and achievement tasks completed by students, whereas qualitative data were obtained from classroom observations, semi-structured interviews, and analysis of instructional documents. Combining these complementary sources of evidence enabled a more comprehensive understanding of current teaching practices while enhancing the credibility and trustworthiness of the research findings.

The mixed-methods approach was considered appropriate because the study sought not only to describe teachers' perceptions and instructional practices but also to interpret the contextual factors influencing the development of mathematical reasoning competency. The integration of quantitative and qualitative evidence allowed the researchers to triangulate findings and identify instructional needs that could not be fully explained by numerical data alone. Such an approach is widely recommended in educational research when investigating complex instructional phenomena involving both measurable outcomes and classroom processes.

According to Creswell and Creswell (2018), mixed-methods research provides a more complete understanding of educational problems by integrating numerical evidence with participants' experiences and contextual information. This methodological perspective is particularly suitable for competency-based mathematics education because it enables researchers to examine instructional practices from multiple perspectives while generating evidence that informs the design of practical educational interventions. Accordingly, the present research design provides a rigorous methodological foundation for analyzing the current situation and developing instructional strategies to strengthen mathematical reasoning competency through the teaching of fractions and fraction operations.

3.2. Participants

The study was conducted in two public primary schools in Bac Ninh Province, Vietnam, namely Quang Thinh Primary School and Kep Primary School. The participants consisted of both teachers and Grade 4 students to provide complementary perspectives on the current implementation of teaching fractions and fraction operations for developing mathematical reasoning competency. A total of 16 Grade 4 teachers participated in the questionnaire survey, including eight teachers from each school. These teachers were directly responsible for teaching Grade 4 mathematics and therefore possessed practical experience in organizing learning activities related to fractions and fraction operations. In addition, 205 Grade 4 students participated in the study, including 115 students from Quang Thinh Primary School and 90 students from Kep Primary School. The student participants represented eight Grade 4 classes across the two schools and completed assessment tasks designed to provide evidence regarding the current development of mathematical reasoning competency.

The participants were selected because they were directly involved in implementing the Grade 4 mathematics curriculum and could provide reliable information regarding both instructional practice and student learning. Teachers contributed evidence concerning their perceptions, instructional approaches, and challenges encountered when developing mathematical reasoning competency through fraction instruction, whereas students' responses provided complementary evidence regarding learning outcomes and classroom implementation. Including both participant groups enabled the study to obtain a more comprehensive understanding of the existing instructional context.

According to Johnson and Onwuegbuzie (2004), involving multiple participant groups in mixed-methods research enhances the credibility of educational findings by enabling researchers to triangulate evidence from different perspectives. Consistent with this methodological principle, the participant selection in the present study ensured that the collected data reflected both teachers' instructional experiences and students' learning situations, thereby providing a reliable empirical foundation for the subsequent development of instructional strategies aimed at strengthening mathematical reasoning competency.

3.3. Data Collection Instruments

Multiple data collection instruments were employed to obtain comprehensive evidence regarding the current status of developing mathematical reasoning competency through the teaching of fractions and fraction operations in Grade 4. The principal instrument was a structured questionnaire administered to Grade 4 teachers. The questionnaire examined teachers' perceptions of the necessity of developing mathematical reasoning competency, their level of attention to reasoning during mathematics instruction, instructional methods currently employed, and the difficulties encountered when organizing learning activities related to fractions and fraction operations. In addition to the questionnaire, student assessment tasks were used to evaluate the current level of mathematical reasoning demonstrated during fraction learning. These quantitative instruments were complemented by classroom observations, semi-structured interviews with teachers, and analysis of relevant instructional documents to provide richer contextual evidence regarding classroom implementation.

The use of multiple instruments enabled the researchers to collect evidence from different sources while reducing the limitations associated with relying on a single data collection method. Questionnaire data provided quantitative information describing teachers' perceptions and instructional practices, whereas classroom observations and interviews offered qualitative insights into how mathematical reasoning was fostered during actual teaching. Student assessment data further complemented these findings by reflecting learners' current reasoning performance within the context of fractions and fraction operations. The integration of these instruments allowed the research findings to present a comprehensive picture of the existing instructional situation.

According to Tashakkori and Teddlie (2010), the combination of quantitative and qualitative instruments strengthens mixed-methods research by allowing researchers to triangulate findings and improve the validity of educational investigations. Consistent with this methodological perspective, the present study employed complementary instruments to ensure that the collected evidence accurately reflected both instructional practices and students' mathematical reasoning, thereby providing a reliable basis for designing instructional strategies.

3.4. Data Collection Procedures

Data collection was conducted systematically in four consecutive stages to ensure that the research objectives were achieved and that the collected evidence accurately reflected the current status of developing mathematical reasoning competency through the teaching of fractions and fraction operations. The first stage involved reviewing relevant literature, curriculum documents, and educational policies concerning competency-based mathematics education and mathematical reasoning. This theoretical review established the conceptual framework for the study and informed the development of the research instruments. Simultaneously, the questionnaire, student assessment tasks, classroom observation protocol, and interview questions were designed and refined before implementation.

The second stage consisted of conducting the field investigation in the two participating primary schools. Questionnaire surveys were administered to Grade 4 teachers, while student assessment tasks were implemented to collect evidence regarding students' mathematical reasoning competency in learning fractions and fraction operations. Classroom observations and semi-structured interviews with teachers were carried out concurrently to obtain additional qualitative information concerning instructional practices, teaching approaches, and challenges encountered during classroom implementation.

The final stage focused on organizing, verifying, and synthesizing the collected data before analysis. Quantitative responses from questionnaires and student assessments were checked for completeness and consistency, whereas qualitative information obtained from observations, interviews, and document analysis was systematically organized according to the research objectives. The use of multiple sources of evidence enhanced the credibility of the findings by allowing the researchers to compare and corroborate information collected from different participants and research methods. Such

systematic data collection procedures are recommended in mixed-methods educational research because they strengthen the reliability, validity, and overall trustworthiness of research findings (McKenney & Reeves, 2021).

3.5. Data Analysis

The collected data were analyzed using both quantitative and qualitative techniques to provide a comprehensive understanding of the current implementation of teaching fractions and fraction operations for developing mathematical reasoning competency. Quantitative data obtained from the teacher questionnaires and student assessment tasks were analyzed using descriptive statistical methods, including frequencies and percentages, to summarize teachers' perceptions, instructional practices, teaching methods, and challenges encountered during classroom implementation. The quantitative findings were organized into tables to facilitate comparison across different aspects of mathematics teaching and to identify the major instructional needs reflected in the survey results.

Qualitative data collected through classroom observations, interviews, and instructional document analysis were examined using content analysis. The qualitative findings were used to explain, verify, and enrich the quantitative results by providing contextual information about teachers' instructional decisions, classroom interactions, and students' opportunities to develop mathematical reasoning during fraction lessons. Integrating qualitative evidence with statistical findings enabled the researchers to interpret the survey results more comprehensively while identifying instructional issues that could not be fully captured through quantitative data alone.

According to Cohen, Manion, and Morrison (2018), combining descriptive statistical analysis with qualitative content analysis enhances the validity and credibility of educational research by allowing researchers to interpret quantitative findings within authentic classroom contexts. Consistent with this methodological perspective, the present study integrated evidence from multiple data sources to generate reliable conclusions regarding the current status of teaching fractions and fraction operations and to provide an empirical foundation for designing instructional strategies that foster mathematical reasoning competency among Grade 4 students.

4. RESULTS

4.1. Teachers' Perceptions of the Importance of Developing Mathematical Reasoning Competency

Table 1. Teachers' Perceptions of the Importance of Developing Mathematical Reasoning Competency

Perceived importance	Frequency (n = 16)	Percentage (%)
Very important	9	56.25
Important	7	43.75
Not important	0	0.00

The survey results indicate a unanimous recognition among participating teachers of the importance of developing mathematical reasoning competency in Grade 4 mathematics. All 16 teachers (100%) considered this competency either very important (56.25%) or important (43.75%), while none regarded it as unnecessary. These findings demonstrate a strong consensus that mathematical reasoning should be regarded as an essential learning outcome in primary mathematics education. Teachers acknowledged that reasoning enables students to explain mathematical ideas, justify solution strategies, and establish logical relationships when learning fractions and fraction operations.

Although teachers expressed highly positive perceptions, evidence from subsequent survey results suggests that this awareness has not yet been fully reflected in classroom practice. Instructional activities continued to focus predominantly on developing basic mathematical thinking and procedural skills, whereas opportunities for students to construct mathematical arguments and justify their reasoning remained comparatively limited. This discrepancy indicates that positive professional beliefs alone are insufficient to ensure effective competency-based instruction unless they are accompanied by practical pedagogical knowledge and appropriate instructional strategies.

The findings are consistent with Jeannotte and Kieran (2017), who argued that mathematical reasoning is a process through which learners generate, justify, validate, and communicate mathematical ideas. Their conceptual model emphasizes that teachers' recognition of the importance of reasoning provides an essential foundation for competency-based mathematics education, but meaningful improvement depends on classroom environments that actively engage students in explanation, justification, argumentation, and reflective mathematical thinking.

4.2. Teachers' Attention to Developing Mathematical Reasoning Competency

Table 2. Teachers' Attention to Developing Mathematical Reasoning Competency

Level of attention	Frequency (n = 16)	Percentage (%)
Frequently	13	81.25
Occasionally	3	18.75
Rarely	0	0.00
Never	0	0.00

The survey results indicate that most participating teachers reported paying regular attention to developing students' mathematical reasoning competency during Grade 4 mathematics lessons. Specifically, 81.25% of teachers stated that they frequently integrated reasoning into instruction, while the remaining 18.75% indicated that they did so occasionally. No respondents selected the options 'rarely' or 'never', suggesting widespread recognition of the importance of reasoning-oriented instruction.

Despite these encouraging responses, classroom observations and interview data revealed that teachers' attention was often concentrated on helping students complete mathematical tasks correctly rather than encouraging them to justify procedures, compare alternative solutions, or construct mathematical arguments. Consequently, opportunities for students to engage in higher-order mathematical reasoning remained relatively limited.

These findings are consistent with Stylianides (2009), who emphasized that mathematical reasoning develops most effectively when classroom instruction deliberately creates opportunities for explanation, justification, argumentation, and critical evaluation. Merely expressing concern for reasoning is insufficient unless instructional activities consistently require students to communicate and defend their mathematical thinking. Therefore, the present findings further support the need for systematic instructional strategies that translate teachers' positive intentions into effective classroom practice.

4.3. Teachers' Priorities and Instructional Approaches for Developing Mathematical Reasoning Competency

Table 3. Teachers' Priorities and Instructional Approaches for Developing Mathematical Reasoning Competency

Dimension	Main finding	Percentage (%)
Instructional priority	Developing basic mathematical thinking	62.50
Instructional priority	Developing mathematical reasoning	18.75
Most frequently used approach	Problem-based teaching	100.00
Frequently used approach	Lecture/explanation	75.00
Constructivist teaching	Used	37.50
Context-based teaching	Used	25.00

The combined findings reveal a noticeable difference between teachers' instructional priorities and the teaching approaches adopted in Grade 4 mathematics classrooms. Most teachers prioritized developing students' basic mathematical thinking, whereas considerably fewer explicitly emphasized mathematical reasoning and justification. This suggests that classroom instruction remained oriented toward procedural competence more than higher-order reasoning.

Regarding instructional approaches, all teachers reported using problem-based teaching, and a large proportion also relied on explanation-based instruction. Constructivist and context-based approaches were implemented less frequently, indicating that although teachers recognized the value of learner-centred instruction, these pedagogical approaches had not yet become common classroom practice. The findings imply that reasoning-oriented teaching requires stronger pedagogical support to move beyond traditional instructional routines.

These findings are consistent with Boaler (2016), who argued that mathematical reasoning develops most effectively in classrooms where students are encouraged to explore ideas, discuss alternative strategies, justify their thinking, and learn through rich mathematical tasks rather than routine procedural exercises. The present results therefore reinforce the need for instructional strategies that systematically embed reasoning opportunities into the teaching of fractions and fraction operations.

4.4. Overall Implications of the Needs Assessment for Instructional Strategy Development

Table 4. Summary of the Main Findings from the Needs Assessment

Aspect	Main finding	Educational implication
Teachers' perceptions	100% recognized the importance of mathematical reasoning	Strong foundation for competency-based reform
Instructional attention	Most teachers reported frequent attention to reasoning	Need to strengthen implementation quality
Instructional priorities	Basic thinking emphasized more than reasoning	Reasoning activities should be embedded systematically
Teaching approaches	Problem-based teaching widely used; learner-centred methods less common	Need practical instructional strategies for reasoning

The combined findings demonstrate that participating teachers possessed a strong awareness of the importance of developing mathematical reasoning competency, and most reported that they regularly considered this objective during mathematics instruction. Nevertheless, analysis across the survey results revealed a clear gap between teachers' educational beliefs and classroom implementation. Although reasoning was widely recognized as an important learning outcome, instructional practice

remained primarily focused on procedural competence and basic mathematical thinking, with relatively limited opportunities for students to justify, explain, and evaluate mathematical ideas.

Another important finding concerns the mismatch between teachers' instructional priorities and the pedagogical approaches adopted in classroom practice. While problem-based teaching was universally reported, constructivist and context-based approaches were implemented much less frequently. This pattern suggests that teachers require more concrete pedagogical support to transform competency-oriented curriculum goals into effective classroom activities, particularly when teaching fractions and fraction operations.

The overall findings are consistent with Lamon (2012), who emphasized that meaningful learning of fractions requires instruction that integrates conceptual understanding, mathematical reasoning, discussion, and justification rather than procedural calculation alone. Therefore, the needs assessment provides a strong empirical basis for designing instructional strategies that systematically foster mathematical reasoning competency while supporting teachers in implementing competency-based mathematics education.

5. DISCUSSION

5.1. Teachers' Positive Perceptions of Mathematical Reasoning Competency and the Gap Between Beliefs and Classroom Practice

The findings of the present study indicate that Grade 4 teachers clearly recognized the importance of developing mathematical reasoning competency through the teaching of fractions and fraction operations. All participating teachers considered this competency to be either important or very important, and most reported that they regularly incorporated reasoning-oriented objectives into their mathematics lessons. These findings suggest that teachers have generally accepted the competency-based orientation promoted by the current mathematics curriculum and acknowledge the educational value of helping students explain mathematical ideas, justify solution strategies, and communicate mathematical thinking. However, the survey also revealed that teachers' positive perceptions were not consistently reflected in classroom implementation. Although reasoning was regarded as an important instructional objective, classroom practice remained largely focused on procedural fluency and basic mathematical thinking, with comparatively limited opportunities for students to construct mathematical arguments or evaluate the validity of different solution strategies.

The present findings are consistent with the study by Sánchez-Mendías, Miñán-Espigares, and Rodríguez-Fernández (2024), which reported that teachers' positive attitudes toward mathematics education contribute significantly to instructional quality but emphasized that effective classroom implementation depends equally on teachers' professional competence and pedagogical preparation. Similarly, Boaler (2016) argued that meaningful mathematical learning occurs when students actively engage in explanation, reasoning, discussion, and reflection rather than merely applying memorized procedures. Positive teacher beliefs therefore represent an important prerequisite for educational

improvement but do not automatically lead to reasoning-oriented classroom practice unless they are accompanied by appropriate instructional knowledge and supportive learning environments.

The present study extends these perspectives by demonstrating that the gap between teachers' educational beliefs and classroom implementation remains evident in the teaching of fractions and fraction operations at the primary level. Although teachers appreciated the importance of mathematical reasoning, they encountered difficulties in designing learning situations that encouraged students to justify mathematical relationships, compare alternative solution strategies, and communicate logical arguments. These findings provide strong empirical support for developing systematic instructional strategies that explicitly integrate reasoning activities into fraction instruction. Such pedagogical support has the potential to transform teachers' positive perceptions into more effective classroom practices while promoting the sustainable development of mathematical reasoning competency among Grade 4 students.

5.2. Challenges in Developing Mathematical Reasoning through Fraction Instruction

The findings of the present study reveal that the challenges encountered by teachers extend beyond the teaching of fraction concepts and are closely associated with the development of mathematical reasoning competency. Although teachers demonstrated strong awareness of the importance of reasoning-oriented instruction, classroom implementation continued to emphasize procedural fluency and basic mathematical thinking rather than explanation, justification, and mathematical argumentation. Teachers also reported that they tended to prioritize simple thinking skills while providing comparatively fewer opportunities for students to explain mathematical relationships or defend their reasoning. This pattern suggests that the principal challenge is not teachers' recognition of educational goals but their capacity to transform these goals into effective classroom practice. Developing mathematical reasoning through fractions requires instructional activities that encourage students to interpret mathematical ideas, compare alternative strategies, justify procedures, and evaluate conclusions rather than merely obtaining correct answers.

These findings are consistent with Lamon (2012), who emphasized that meaningful learning of fractions depends on students' conceptual understanding and mathematical reasoning rather than memorizing computational rules. Lamon argued that instruction should engage learners in interpreting fraction concepts through multiple representations, discussing mathematical relationships, and justifying solution strategies to overcome persistent misconceptions associated with fractions. Similarly, Stylianides and Stylianides (2009) highlighted that the development of mathematical reasoning requires carefully structured instructional environments in which students gradually progress from intuitive explanations toward logically justified mathematical arguments. Such learning experiences enable students to evaluate mathematical claims critically and communicate their reasoning with increasing precision.

The present study reinforces these international perspectives by demonstrating that similar instructional challenges remain evident in Grade 4 mathematics classrooms. Although teachers

frequently employed problem-based teaching, opportunities for students to construct mathematical arguments and engage in reflective reasoning were still relatively limited. These findings suggest that improving the teaching of fractions requires systematic instructional strategies that explicitly integrate reasoning into every stage of classroom instruction. By providing practical guidance for designing reasoning-oriented learning activities, such strategies may help teachers bridge the gap between competency-based curriculum expectations and classroom practice, thereby promoting more effective development of mathematical reasoning competency among primary school students.

5.3. Implications for Instructional Strategy Development

The findings of the present study provide a clear empirical foundation for the development of instructional strategies that strengthen mathematical reasoning competency through the teaching of fractions and fraction operations. Although teachers demonstrated strong awareness of the importance of mathematical reasoning and frequently reported integrating competency-oriented objectives into their teaching, the survey results revealed important limitations in classroom implementation. Teachers primarily emphasized basic mathematical thinking and procedural problem solving, whereas opportunities for students to construct mathematical arguments, justify mathematical relationships, and critically evaluate solution strategies remained comparatively limited. These findings suggest that improving mathematical reasoning requires not only curriculum reform but also systematic instructional guidance that supports teachers in translating educational expectations into effective classroom practice.

The present findings are consistent with McKenney and Reeves (2021), who argued that educational design research should generate practical instructional solutions grounded in authentic classroom problems and empirical evidence. Rather than proposing general pedagogical recommendations, instructional strategies should be developed through a systematic process that integrates theoretical foundations, classroom realities, and iterative refinement. Likewise, Plomp and Nieveen (2013) emphasized that effective instructional design must respond directly to identified educational needs while remaining feasible for implementation within everyday classroom contexts. These perspectives strongly support the methodological approach adopted in the present study, in which the proposed instructional strategies were developed from the needs identified through teacher surveys, classroom observations, and interviews.

The present study therefore extends previous research by demonstrating that the design of instructional strategies should explicitly integrate mathematical reasoning into every stage of fraction instruction. Rather than treating reasoning as an additional learning objective, instructional activities should encourage students to explain mathematical ideas, compare alternative solution strategies, justify mathematical conclusions, and reflect critically on their own reasoning throughout the learning process. Such instructional strategies can help teachers transform positive educational beliefs into effective classroom practice while promoting deeper conceptual understanding of fractions and strengthening students' mathematical reasoning competency. Consequently, the proposed instructional

framework provides practical guidance for teachers, curriculum developers, and teacher educators seeking to implement competency-based mathematics education more effectively in primary schools.

5.4. Educational Implications for Competency-Based Mathematics Teaching

The findings of this study have important implications for improving competency-based mathematics education in primary schools. Although teachers demonstrated positive perceptions of mathematical reasoning competency and acknowledged its importance in the teaching of fractions and fraction operations, the survey results revealed that classroom implementation remained largely oriented toward procedural instruction. Students were provided with relatively few opportunities to explain mathematical relationships, justify solution strategies, compare different approaches, or evaluate mathematical arguments. These findings suggest that achieving the goals of competency-based mathematics education requires not only curriculum reform but also systematic changes in instructional practice that position mathematical reasoning as a central learning objective throughout the teaching process.

The present findings are consistent with Darling-Hammond et al. (2020), who argued that meaningful learning occurs when students actively construct knowledge through inquiry, reasoning, collaboration, and reflection within supportive classroom environments. Such learning experiences enable students to develop transferable competencies rather than simply acquire isolated knowledge and procedural skills. Likewise, Niss and Højgaard (2019) emphasized that mathematical competencies should be cultivated through authentic mathematical activities that require learners to reason, communicate, represent, and apply mathematical ideas in meaningful contexts. These perspectives reinforce the view that mathematical reasoning should become an integral component of everyday mathematics instruction rather than an additional or optional classroom activity.

Building upon these international perspectives, the present study demonstrates that carefully designed instructional strategies can effectively bridge the gap between curriculum expectations and classroom implementation. By guiding teachers in organizing reasoning-oriented learning activities, encouraging mathematical explanation and justification, and integrating conceptual understanding with problem solving throughout fraction instruction, such strategies can enhance both instructional quality and students' mathematical reasoning competency. Beyond the teaching of fractions, the proposed instructional framework also provides practical implications for teacher professional development, curriculum implementation, and instructional design in primary mathematics education. Consequently, the findings of this study offer valuable evidence for strengthening competency-based mathematics teaching while supporting the sustainable development of mathematical reasoning among primary school students.

6. CONCLUSION

This study investigated the current status of developing mathematical reasoning competency through the teaching of fractions and fraction operations in Grade 4 primary mathematics and established an empirical foundation for designing instructional strategies aligned with competency-based

mathematics education. By integrating questionnaire surveys, classroom observations, interviews, and document analysis, the study provided a comprehensive understanding of teachers' perceptions, instructional practices, and challenges associated with fostering mathematical reasoning in primary classrooms. The findings demonstrated that participating teachers clearly recognized the educational importance of developing mathematical reasoning competency and generally expressed positive attitudes toward competency-oriented mathematics instruction. Nevertheless, the results also revealed a noticeable discrepancy between teachers' professional beliefs and their classroom practices. Although teachers frequently emphasized the importance of reasoning, instruction continued to focus primarily on procedural fluency and basic mathematical thinking, while opportunities for students to explain mathematical ideas, justify solution strategies, construct mathematical arguments, and evaluate mathematical conclusions remained comparatively limited.

The study further demonstrates that improving students' mathematical reasoning competency requires more than curriculum reform or increased teacher awareness. Effective competency-based mathematics education depends on systematically designed instructional strategies that integrate reasoning into every stage of teaching and learning. In the context of fractions and fraction operations, reasoning should not be regarded as a supplementary instructional objective but as an integral component of conceptual understanding, problem solving, mathematical communication, and reflective thinking. The empirical findings therefore provide strong justification for developing instructional strategies that support teachers in organizing authentic mathematical activities, promoting classroom discussion, encouraging mathematical justification, and creating learning environments in which students actively construct and communicate mathematical reasoning.

The present study contributes to both theory and educational practice by providing empirical evidence concerning the current implementation of reasoning-oriented mathematics teaching in primary schools while establishing a practical basis for instructional design. The findings offer useful implications for curriculum implementation, teacher professional development, and competency-based mathematics instruction, particularly in the teaching of fractions and fraction operations. However, the study was conducted in two primary schools within a single province, and the findings should therefore be interpreted within this educational context. Future research should examine the effectiveness of the proposed instructional strategies across more diverse educational settings, investigate their long-term impact on students' mathematical reasoning competency, and explore how digital technologies and innovative pedagogical approaches can further strengthen reasoning-oriented mathematics instruction. Overall, the study provides a coherent foundation for enhancing the quality of primary mathematics education by supporting the systematic development of mathematical reasoning competency through meaningful instruction in fractions and fraction operations.

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