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**THE IMPACTS OF MATHEMATICS TEACHING THROUGH PROBLEM-SOLVING CONTEXTS ON SECONDARY SCHOOL STUDENTS' PROBLEM-SOLVING PERFORMANCE AND REPRESENTATION USE**

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**ABSTRACT**

This paper focused on secondary students' problem-solving performance and representation use as a result of an instructional intervention. The instructional intervention aims to supplement current efforts to enhance students' problem-solving performance and number of representations used to solve word problems. Quasi- experimental research design was employed for the study. This study was conducted with a sample of 58 Secondary School (SS II) Students from two senior secondary schools in Damaturu Local Government Area, Yobe State of Nigeria. These schools were randomly assigned to intervention group and comparison group through intact class sampling procedure. The content validity and reliability of the instruments were established by the use of Mathematics Performance Test (MPT) designed by the researcher which was also vetted by the experts in the field (mathematics education). The instruments were also trial tested and the reliability coefficient for the pre-and posttest of 0.79 and 0.88 were obtained. Mean, Standard deviation, Chi-square and t-test statistics were used for data analysis to answer the research questions stated in the study. The findings of the study indicated that the intervention group had a positive effect or impacts on students' problem-solving performance and number of representations used on the posttest whereas the comparison group experienced no changes. It also indicated that intervention students solved more word problems and used more representations on the posttest than their peers. Hence, recommendations and conclusion were advanced in respect of the study.

**KEYWORDS:** Students, Mathematics teaching, Problem solving, Performance, Representation use.

**1.1 INTRODUCTION**

Problem solving as viewed from a mathematics education perspective is the process of interpreting a situation mathematically, which usually involves several iterative cycles of expressing, testing and revising mathematical interpretations – and of sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics (Lesh and Zawojewski ,2007).

Pittman (2006) defines problem solving as a complex activity that requires individuals to maintain focus and both rationally and effectively proceed through the problem. Students' ineffective problem-solving behaviors and disengagement in the process is exacerbated by teacher-directed instruction that frequently uses too many exercises and not enough problems.

One way to foster students' success in the problem-solving process is to provide them with frequent opportunities to engage in problem solving in a student-centered environment that scaffolds students to successfully complete each stage of the process (Verschaffel & De Corte, 1997; Verschaffel et al., 1999).

Many studies have demonstrated that when daily mathematics instruction is integrated or supplemented with problem-solving activity, it enhances students' problem-solving capabilities (Charles & Lester, 1984; Sigurdson, Olson, & Mason, 1994; Verschaffel et al., 1999). Moreover, there is some evidence that students' learning in classroom environments where problem solving is a regular part of mathematics instruction outperform their peers in traditional learning environments on mathematics achievement tests. Success on problem-solving and achievement measures is also influenced by the degree to which students are supported to gain facility with representations and procedures.

According to Lambdin (2003), problem-solving practice during mathematics instruction enhances students' mathematical understanding and in turn, well-developed mathematical understanding supports individuals to become more efficient and effective problem solvers.

Lampert (1990) examined teaching episodes individually and then looked for patterns of change in students' outcomes. As a result of her instruction, students developed effective problem-solving behaviors, indicated more positive feelings about doing mathematics, and learned to work collaboratively to solve challenging problems.

Effectively engaging in the problem-solving process requires individuals to maintain their focus on a number of factors and work through each stage (Pape, 2004; Verschaffel et al., 2000)

Representations are absolutely necessary for any mathematical activity to occur because mathematics typically uses sequences of symbolic characters that convey shared meanings among individuals (Kaput & Educational Technology Center, 1989). They provide a means to link two or more configurations of an idea or concept (Goldin, 2002). In the context of word problems, students create representations that "(a) reproduce the action of a story problem; (b) strip away the context, attending only to numerical aspects of the problem; or (c) combine some of both approaches" (Smith, 2003, p. 263).

Representation use is a critically important element of solving problems. Algorithms have been and continue to be a focus in many mathematics classrooms (Boaler, 2008). They are important tools for solving mathematics problems and should be part of mathematics instruction (National Mathematics Advisory Panel, 2008), but focusing mathematics instruction on learning algorithms does not orient students to determine the essential parts of a problem's situation or enhance their problem-solving performance (Thompson, 2008). Instruction that allows students to manipulate tasks into more manageable or useful representations and employ a variety of representations and procedures facilitates children's development of mathematical proficiency (Hiebert, 2003; Packer, 2003; Van de

Walle, 2003). Therefore, this present study is aimed at investigating secondary school students' problem-solving performance and representation use.

## 1.2 OBJECTIVES OF THE STUDY

The objective of this study was to investigate the impacts/effects of instructional intervention on students' problem-solving performance when solving word problems. Creating a supportive instructional context that used word problems as the focal activity was intended to support students' opportunities for learning mathematics content and procedures. Specifically, the study sought to determine whether:

- (i) Intervention influence students' performance on a test of word problems.
- (ii) Intervention influence the total number of representations students use on a word problem test.
- (iii) There is a relationship between intervention status and students' use of non symbolic representations on the problem-solving posttest.

## 1.3 RESEARCH QUESTIONS

The following research questions were stated to guide the study:

- (i) Does the intervention influence students' performance on a test of word problems?
- (ii) Does the intervention influence the total number of representations students use on a word problem test?
- (iii) Is there a relationship between intervention status and students' use of non symbolic representations on the problem-solving posttest?

## 2. MATERIALS AND METHODS

### 2.1 Introduction

This section dealt with the steps or procedures taken in carrying out the study. The research design was adopted and equally justified its selection as well as instruments used for data collection and also spelt out the target or access population from which the sample of participants were drawn

### 2.2 Research Design

This study adopted non equivalent quasi-experimental research design (Gall et al., 2007). Quasi-experimental designs are necessary when there is inability to randomly assign individual respondents to each group, thus there are multiple potential threats to internal validity (Gall et al., 2007).

### 2.3 Research Population and Sample

The population of the study comprised 120 students from two secondary schools in Damaturu Local Government Area of Yobe State, Nigeria. Out of this number, a total of 58 students in SSII formed the sample for the study through random sampling techniques.

### 2.4 Research Instruments

Two instruments used to facilitate data collection for this study include a Problem-Solving Pretest and Problem-Solving Posttest Mathematics Performance Test (MPT) designed by the researcher (instructor). The two problem-solving measures were designed to capture students’ problem-solving performance and their representation use during problem solving. The pre- and posttest had five items each and were administered during students’ regular mathematics classes respectively. Items were presented individually on separate sheets of paper. Participants were asked to solve each problem and provide additional solution methods, if known. While coding and scoring protocols were used to score students’ performance as correct or incorrect measures in the area of representation use.

**2.5 Validity and Reliability of Instruments**

For validity of the instruments, face validity was used. The instruments were given to experts who are knowledgeable in the field of Mathematics Education to certify that the instruments measured what is purported to measure.

To ensure the reliability of the instruments, a trial testing of the instruments was carried out of 30 students in secondary schools, the instruments was later re-administered to the same students after one week and the reliability coefficient fors pre-test and posttest of 0.79 and 0.88 were obtained. This is to reduce potential problems with the wording of the items which were the impetus for test-re-test.

**2.6 Data Analysis**

The pretest and posttest scores gathered/collected were analyzed by using mean, standard deviation, chi-square and t-test statistics.

**3. ANALYSIS AND RESULTS PRESENTATION**

**Table 1: Pretest and Posttest Mean Scores and Standard Deviations of the Intervention and Comparison Related to Problem Solving Performance, and Representation use.**

		Intervention Group (A)		Comparison Group (B).			
Factors	Mean(x)	SD	Mean (x)	SD	t-value	p-value	Dec.
Pretest Performance	2.23	1.17	1.66	1.51	0.52	0.61	
Total Representation Use	2.78	1.73	1.66	1.92	0.22	0.83	
Post-test Performance	2.83	1.30	1.73	1.28	2.65	0.02	Sig.

Total Representation Use	3.50	1.69	2.20	1.64	2.60	0.02	Sig.
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**Key: For Group (A), N= 18, Group(B), N= 40**

To answer the first research question, results from a paired samples t-test indicated that the students in the intervention group had better performance on the posttest than the pretest, t-value= 2.65, p-value = 0.02, whereas their peers did not improve, t-value = 0.52, p-value = 0.61. The intervention group’s problem-solving growth is approximately 17% greater than the finding related to annual gains in mathematics achievement (Bloom et al., 2008).

Also from table 1, the analyses related to the second research question revealed that intervention group (respondents) employed more representations on the posttest compared to the pretest, t-value = 2.60, and p-value =0.02 respectively. This result shows a slight improvement over the average annual gain in mathematics achievement, approximately 2%. There were significant changes in their peers’ total representation use, t-value = 0.22, p-value = 0.83. Thus, students in the intervention group improved their performance and used more representations on the posttest than the pretest, but their peers in the comparison group did not demonstrate changes related to their problem solving.

**Table 2: Group Means and Standard Deviations of the type of Representation Used on the Posttest**

Representation	Intervention Group (A)		Comparison Group (B)	
	Mean (x)	SD	Mean (x)	SD
Symbolic	2.78	1.35	1.45	1.15
Tabular	0.17	0.38	0.23	0.42
Pictorial	0.44	0.62	0.48	0.64
Verbal	0.0	0.0	0.0	0.0
Mixed	0.11	0.32	0.08	0.27

**Key: For Group (A), N= 18, Group (B), N= 40**

Table 2 provides the means and standard deviations of each group’s posttest representation use. A chi-square test was used to answer the third research question: whether there is a relationship between intervention status and students’ use of non symbolic representations on the posttest (i.e pictorial, symbolic, verbal, tabular, and mixed). There is no relationship between intervention status and students’ non symbolic representation use,  $\chi^2(1) = 0.62, p = .43$ . These results suggest that respondents (participants) in the intervention group were not associated with use of non symbolic representations on the posttest.

## 4. SUMMARY OF THE FINDINGS

The following submissions are the major summary of the findings for this study:

- (i) Students in the intervention group had higher scores on the posttest than the pretest. They also used more representations on the posttest than they did on the pretest.
- (ii) There was a statistically significant difference in students' performance and the total number of representations used on the posttest, favoring the intervention group.
- (iii) There was no relationship between intervention status and use of non symbolic representations on the posttest.

## 5. DISCUSSION OF RESULTS

The primary concern of this study was to examine students' problem-solving performance and representation use as a result of an instructional intervention. This investigation was guided by prior research indicating that students learning mathematics through problem-solving contexts might demonstrate enhanced problem-solving performance (e.g., Charles & Lester, 1984; Sigurdson et al., 1994; Verschaffel et al., 1999) as well as improved representation use while problem solving (Klein et al., 1998) compared to their peers experiencing their typical instruction. The open, complex, and realistic word problems were critical to the instructional intervention. The instructors in the intervention classroom aimed to maintain a student-centered, discourse-rich classroom with the goal of supporting students' problem solving as well as their mastery of concepts and procedures. Data related to several predictor variables were collected and later investigated in relation to students' outcomes.

The intervention group improved their problem-solving performance after the intervention whereas the comparison group did not. These findings are consistent with Sigurdson and Olson (1992) and provide information about the impact of problem-solving interventions. The effect size ( $d = 0.48$ ) associated with the pre- and posttest differences gives an indication of the intervention's positive effect. The comparison group respondents' problem-solving performance did not increase significantly between test administrations. The results indicated that the intervention supported students' problem-solving performance.

Also, the intervention supported participants (respondents) from the intervention group to use more representations on the posttest than the pretest. More precisely, they provided approximately one additional representation on the posttest. The comparison group participants did not use more representations after the usual instruction. The effect size ( $d = .42$ ) suggests that the intervention somewhat impacted students' representation use. The statistic provides an initial result, which could be confirmed by future research. Prior research has shown that teaching through problem-solving contexts supports students' ability to create appropriate mathematical models to solve word problems (Verschaffel & De Corte, 1997). The evidence from the present study and Verschaffel and De Corte tell a similar story: Using open, complex, and realistic problems with adolescents in student-centered

discourse-rich instructional contexts enhances the number of representation used to solve problems whereas traditional instruction does not.

This finding related to respondents' representation use from the present study is also inline with explorations with younger children in the Netherlands (Klein et al., 1998). Second-grade students who experienced instruction that encouraged them to utilize a variety of representations were more likely to successfully employ more representations to solve exercises and problems than their peers in comparison classrooms. Drawing on evidence from these three studies, there is a growing body of evidence indicating that student-centered, discourse-rich instruction positively impacts students' use of representations on problem-solving tasks.

For research question three, there is no significant relationship between non symbolic representation use on the posttest and intervention status. Instruction in the intervention classroom typically encouraged students to think about whether there were alternate representations and procedures to solve a problem. This non significant finding may be influenced by several factors including students' ability to use non symbolic representations (Preston & Garner, 2003) as well as their perceptions related to employing non symbolic representations to solve word problems (Bostic&Pape, 2010; Herman, 2007). Herman noticed that Algebra students perceived pictorial and tabular representations as backup methods to verify a symbolic-oriented representation. Further research is necessary to explain why no significant relationship was produced by the intervention.

## 6. CONCLUSION

The concern of this study was to explore whether the intervention enhanced students' performance and representation use on word problem tests. Conclusively, intervention participants (respondents) had better problem-solving performance and representation use between test administrations. Similarly, they also performed better than their peers in the comparison group. There was no relationship between non symbolic representation use and intervention status. This investigation demonstrates that it is possible to teach mathematics from the Standards through problem-solving contexts and in ways that develop effective mathematical practices in Nigeria and the World at large.

## 7. RECOMMENDATIONS

Based on the findings of this study, it was recommended that:

- (i) Systematic approach to teaching and learning should be adopted by mathematics teachers for effective teaching and learning process. This will facilitate students' problem solving performance as well as the representation use.
- (ii) Workshops, conferences, seminars (capacity building programmes) and induction training courses for mathematics teachers should be organized by the Government to incorporate the new teaching strategy to improve mathematics teaching in Damaturu L.G.A of Yobe State and Nigeria as an entity. This provides teachers with opportunity to develop knowledge and



skills and widen or broaden their teaching methods, thereby creating better learning opportunities for students (learners).

## REFERENCES

- Bloom, H., Hill, C., Black, A., & Lipsey, M. (2008). Performance trajectories and performance gaps as achievement effect-size benchmarks for educational interventions. Retrieved from MDRC website <http://www.mdrc.org/publications>
- Bostic, J., & Pape, S. (2010). Examining Students' Perceptions of Two Graphing Technologies and Their Impact on Problem Solving. *Journal of Computers in Mathematics and Science Teaching*, 29, 139-154.
- Gall, M., Gall, J., & Borg, W. (2007). *Educational research: An introduction* (8th ed.). Boston: Pearson.
- Goldin, G. (2002). Representation in mathematical learning and problem solving. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 197-218). Mahwah, NJ: Erlbaum.
- Herman, M. (2007). What students choose to do and have to say about use of multiple representations in college algebra. *Journal of computers in mathematics and science teaching*, 26, 27-54.
- Hiebert, J. (2003). Signposts for teaching mathematics through problem solving. In F. Lester, Jr. & R. Charles (Eds.), *Teaching mathematics through problem solving: Prekindergarten-grade 6* (pp. 51-61). Reston, VA: National Council of Teachers of Mathematics.
- Kaput, J., & Educational Technology Center. (1989). Linking representations in the symbol systems of algebra. In S. Wagner & C. Kieran (Eds.), *Research issues in the learning and teaching of algebra* (pp. 167-194). Reston, VA: National Council of Teachers of Mathematics.
- Klein, A., Beishuizen, M., & Treffers, A. (1998) The empty number line in Dutch second grades: Realistic versus gradual program design. *Journal for Research in Mathematics Education*, 29, 443-464.
- Lambdin, D. (2003). Benefits of teaching through problem solving. In F. Lester, Jr., & R. Charles (Eds.), *Teaching mathematics through problem solving: Prekindergarten – grade 6* (pp. 3-13). Reston, VA: National Council of Teachers of Mathematics.
- Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27, 29-63.
- Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763-804). Charlotte, NC: Information Age Publishing.
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Retrieved from the United States Department of Education website <http://www.ed.gov/MathPanel>
- Pittman, M. (2006). Multiple solution strategies in the middle school mathematics classroom: The role of the teacher. ProQuest Dissertations and Theses database. (UMI No. 3219047)



- Preston, R., & Garner, A. (2003). Representation as a vehicle for solving and communicating. *Mathematics Teaching in the Middle School*, 9, 38-43
- Packer, A. (2003). Making mathematics meaningful. In B. Madison & L. Steen (Eds.), *Quantitative literacy: Why numeracy matters for schools and colleges*. Princeton, NJ: National Council on Education and the Disciplines.
- Hiebert, J. (2003). Signposts for teaching mathematics through problem solving. In F. Lester, Jr. & R. Charles (Eds.), *Teaching mathematics through problem solving: Prekindergarten-grade 6* (pp. 51-61). Reston, VA National Council of Teachers of Mathematics.
- Sigurdson, S., & Olson, A. (1992). Teaching mathematics with meaning. *Journal of Mathematical Behavior*, 11, 37-57.
- Sigurdson, S., Olson, A., & Mason, R. (1994). Problem solving and mathematics learning. *Journal of Mathematical Behavior*, 13, 361-388.
- Smith, S. (2003). Representation in school mathematics: Children's representations of problems. In Thompson, P. (2008). On professional judgment and the National Mathematics Advisory Panel report: Curricular content. *Educational Researcher*, 37 582-587.
- Verschaffel, L., & De Corte, E. (1997). Teaching realistic mathematical modeling in the elementary school: A teaching experiment with fifth graders. *Journal for Research in Mathematics Education*, 28, 577-601.
- Van de Walle, J. (2003). Designing and selecting problem-based tasks. In F. Lester, Jr. & R. Charles (Eds.), *Teaching mathematics through problem solving: Prekindergarten-grade 6* (pp. 67-80). Reston, VA: National Council of Teachers of Mathematics.
- Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999). Learning to solve mathematical application problems: A design experiment with fifth graders. *Mathematical Thinking and Learning*, 1, 195-229.